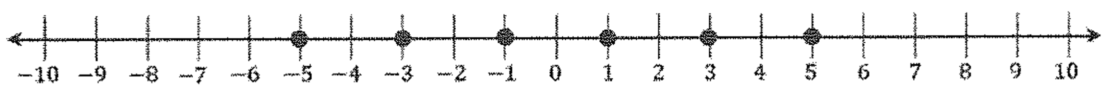
Homework Helpers

Grade 6
Module 3

G6-M3-Lesson 1: Positive and Negative Numbers on the Number Line—Opposite Direction and Value

1. Draw a number line, and create a scale for the number line in order to plot the points $-1, 3$, and 5.
   a. Graph each point and its opposite on the number line.
   b. Explain how you found the opposite of each point.

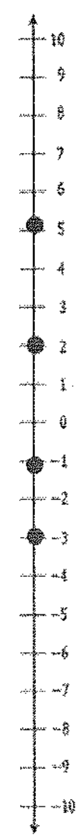


I know that opposite numbers are the same distance from zero, except in opposite directions.

To graph each point, start at zero, and move right or left based on the sign and number (to the right for a positive number and to the left for a negative number). To graph the opposites, start at zero, but this time move in the opposite direction the same number of times.

2. Kip uses a vertical number line to graph the points $-3, -1, 2$, and 5. He notices -3 is closer to zero than -1. He is not sure about his diagram. Use what you know about a vertical number line to determine if Kip made a mistake or not. Support your explanation with a vertical number line diagram.

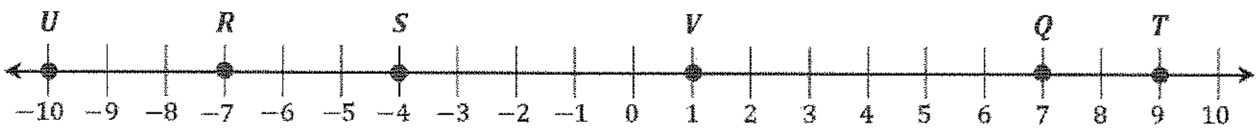
   Kip made a mistake because -3 is less than -1, so it should be farther down the number line. Starting at zero, negative numbers decrease as we look farther below zero. So, -1 lies before -3 since -1 is 1 unit below zero, and -3 is three units below zero.



I know that values increase as I look up on a vertical number line and decrease as I look down. Numbers above zero are positive, and numbers below zero are negative.

3. Create a scale in order to graph the numbers -10 through 10 on a number line. What does each tick mark represent?

Each tick mark represents one unit.



4. Choose an integer between -4 and -9. Label it R on the number line created in Problem 3, and complete the following tasks.

Answers will vary. Answers a-e reflect the student choice of -7. -7 is between -4 and -9.

a. What is the opposite of R? Label it Q.

The opposite of -7 is 7.

b. State a positive integer greater than Q. Label it T.

A positive integer greater than 7 is 9 because 9 is farther to the right on the number line.

c. State a negative integer greater than R. Label it S.

A negative integer greater than -7 is -4 because -4 is farther to the right on the number line.

d. State a negative integer less than R. Label it U.

A negative integer less than -7 is -10 because -10 is farther to the left on the number line.

e. State an integer between R and Q. Label it V.

An integer between -7 and 7 is 1.

5. Will the opposite of a positive number always, sometimes, or never be a positive number? Explain your reasoning.

The opposite of a positive number will never be a positive number. For two nonzero numbers to be opposite, zero has to be between both numbers, and the distance from zero to one number has to equal the distance between zero and the other number.

6. Will the opposite of zero always, sometimes, or never be zero? Explain your reasoning.

   The opposite of zero will always be zero because zero is its own opposite.

7. Will the opposite of a number always, sometimes, or never be greater than the number itself? Explain your reasoning. Provide an example to support your reasoning.

   The opposite of a number will sometimes be greater than the number itself because it depends on the given number. The opposite of a negative number is a positive number, so the opposite will be greater. But, the opposite of a positive number is a negative number, which is not greater. Also, if the number given is zero, then the opposite is zero, which is never greater than itself.

G6-M3-Lesson 2: Real-World Positive and Negative Numbers and Zero

1. Express each situation as an integer in the space provided.

   a. A gain of 45 points in a game

   45

   > I know words that imply a positive magnitude include "gain" and "deposit." Words that imply a negative magnitude include "fee charged," "below zero," and "loss."

   b. A fee charged of $3

   -3

   c. A temperature of 20 degrees Celsius below zero

   -20

   d. A 35-yard loss in a football game

   -35

   e. A $15,000 deposit

   $15,000$

2. Each sentence is stated *incorrectly*. Rewrite the sentence to correctly describe each situation.

   a. The temperature is -20 degrees Fahrenheit below zero.

   The temperature is 20 degrees Fahrenheit below zero. Or, the temperature is -20 degrees Fahrenheit.

b. The temperature is -32 degrees Celsius below zero.

The temperature is 32 degrees Celsius below zero. Or, the temperature is -32 degrees Celsius.

I know that magnitude can be determined by the use of language in a problem. "Below zero" means that the number being referenced will be negative, so I do not need to use a negative sign. Or, if I choose to use a negative sign, I do not need the term "below zero" because the number is already negative.

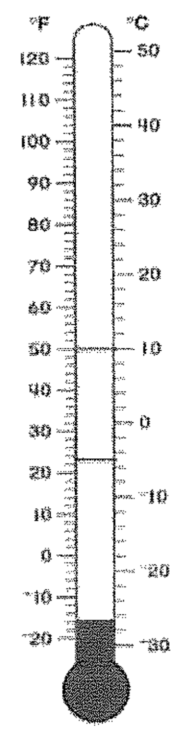
For Problems 3–5, use the thermometer to the right.

3. Mark the integer on the thermometer that corresponds to the temperature given.

   a. $50°\text{F}$

   b. $-5°\text{C}$

The Fahrenheit scale is on the left of the thermometer, and the Celsius scale is on the right. I need to mark the integers on the correct scale.



4. The melting point of steel is $1{,}510°\text{C}$. Can this thermometer be used to record the temperature of the melting point of steel? Explain.

   The melting point of steel cannot be represented on this thermometer. The highest this thermometer gauges is $50°\text{C}$. $1{,}510°\text{C}$ is a much larger value.

5. Natalie shaded the thermometer to represent a temperature of 15 degrees below zero Celsius, as shown in the diagram. Is she correct? Why or why not? If necessary, describe how you would fix Natalie's shading.

   Natalie is incorrect. She did shade in $-15°$ but on the wrong scale. The shading represents $-15°\text{F}$, instead of $-15°\text{C}$. To fix Natalie's mistake, the shading must be between -10 and -20 on the Celsius scale.

G6-M3-Lesson 3: Real-World Positive and Negative Numbers and Zero

1. Write an integer to match the following descriptions.

a.	A debit of $50	-50
b.	A deposit of $125	125
c.	$5,600$ feet above sea level	$5,600$
d.	A temperature increase of $50°\text{F}$	50
e.	A withdrawal of $125	-125
f.	$5,600$ feet below sea level	$-5,600$

I know words that describe positive integers include "deposit," "above sea level," and "increase." Words that describe negative integers include "debit," "withdrawal," and "below sea level."

For Problems 2 and 3, read each statement about a real-world situation and the two related statements in parts (a) and (b) carefully. Circle the correct way to describe each real-world situation; *possible answers include either (a), (b), or both (a) and (b).*

2. A shark is 500 feet below the surface of the ocean.

   a. The depth of the shark is 500 feet from the ocean's surface.

   b. The whale is -500 feet below the surface of the ocean.

To represent a negative integer, I know I can use a negative sign or vocabulary that determines magnitude, but not both.

3. Carl's body temperature decreased by $3°\text{F}$.

   a. Carl's body temperature dropped $3°\text{F}$.

   b. The integer -3 represents the change in Carl's body temperature in degrees Fahrenheit.

The word "dropped" tells me the integer is negative. A "decrease" also tells me the integer is negative. I know that -3 represents a negative integer and the change in the temperature, so both of these examples are correct.

4. A credit of $45 and a debit of $50 are applied to your bank account.

   a. What is the appropriate scale to graph a credit of $45 and a debit of $50? Explain your reasoning.

   Because both numbers are divisible by 5, an interval of 5 is an appropriate scale on a number line.

   b. What integer represents "a credit of $45" if zero represents the original balance? Explain.

   45; a credit is greater than zero, and numbers greater than zero are positive numbers.

   c. What integer describes "a debit of $50" if zero represents the original balance? Explain.

   −50; a debit is less than zero, and numbers less than zero are negative numbers.

   d. Based on your scale, describe the location of both integers on the number line.

   If the scale is created with multiples of 5, then 45 would be 9 units to the right (or above) zero, and −50 would be 10 units to the left (or below) zero.

   e. What does zero represent in this situation?

   Zero represents no change being made to the account balance. No amount is either added to or subtracted from the account.

G6-M3-Lesson 4: The Opposite of a Number

1. Find the opposite of each number, and describe its location on the number line.

   a. -4

   The opposite of -4 is 4, which is 4 units to the right of (or above) zero if the scale is one.

   b. 8

   The opposite of 8 is -8, which is 8 units to the left of (or below) zero if the scale is one.

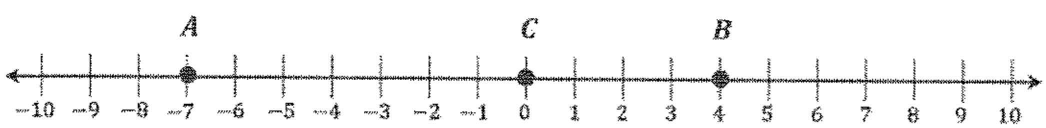
> I know the opposite of any integer is on the opposite side of zero at the same distance. Since -4 is 4 units to the left of zero, then 4 units to the right of zero is 4. The opposite of -4 is 4. The opposite of 8 has to be -8 because -8 is the same distance from zero, just to the left.

2. Write the opposite of each number, and label the points on the number line.

   a. Point A: the opposite of 7 $\quad$ -7

   b. Point B: the opposite of -4 $\quad$ 4

   c. Point C: the opposite of 0 $\quad$ 0



> 7 is located 7 units to the right of zero, so the opposite of 7 must be 7 units to the left of zero. I know -4 is located 4 units to the left of zero, so its opposite has to be 4 units to the right of zero. I also know that zero is its own opposite.

EUREKA MATH™

3. Study the first example. Write the integer that represents the opposite of each real-world situation. In words, write the meaning of the opposite.

   a. An atom's negative charge of -9

      9, an atom's positive charge of 9

   b. A deposit of $15

      −15, a withdrawal of $15

   c. $2,500$ feet below sea level

      2,500, 2,500 feet above sea level

   d. A rise of $35°C$

      −35, a decrease of 35°C

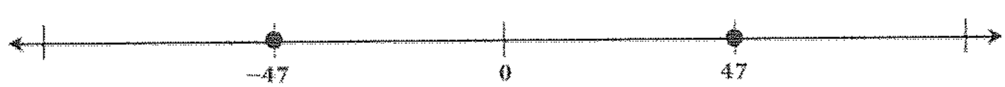
   e. A loss of 20 pounds

      20, a gain of 20 pounds

> I know the following opposites:
> negative/positive
> deposit/withdrawal
> below sea level/above sea level
> rise/decrease
> loss/gain
> Using these opposites, I can determine the opposite of the integers in the situations.

4. On a number line, locate and label a credit of $47 and a debit for the same amount from the bank. What does zero represent in this situation?

   Zero represents no change in the balance.



> At the beginning of my transactions, my bank account is a fixed number. If I do not change it, then the change is represented with zero. If I have a credit of 47, I know that that is an increase and falls to the right of zero. If I have a debit of 47, I know that that is a decrease and falls to the left of zero.

G6-M3-Lesson 5: The Opposite of a Number's Opposite

1. Read each description carefully, and write an equation that represents the description.

   a. The opposite of negative six

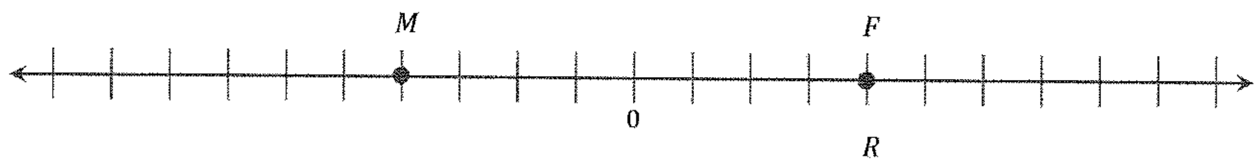
   $-(-6) = 6$

   b. The opposite of the opposite of thirty-five

   $-(-(35)) = 35$

The opposite of a negative number is positive because it is on the opposite side of zero on the number line. The opposite of the opposite of a positive number is positive because the first opposite is on the left side of zero on the number line. The next opposite is to the right of zero.

2. Carol graphed the opposite of the opposite of 4 on the number line. First, she graphed point F on the number line 4 units to the right of zero. Next, she graphed the opposite of F on the number line 4 units to the left of zero and labeled it M. Finally, she graphed the opposite of M and labeled it R.



   a. Is her diagram correct? Explain. If the diagram is not correct, explain her error, and correctly locate and label point R.

   Yes, her diagram is correct. It shows that F is 4 because it is 4 units to the right of zero. The opposite of 4 is -4, which is point M (4 units to the left of zero). The opposite of -4 is 4, so point R is 4 units to the right of zero.

   b. Write the relationship between the points.

   F and M

   They are opposites.

   M and R

   They are opposites.

   F and R

   They are the same.

I see that points M and F are exactly the same distance from zero, just in opposite directions, so they are opposites. M and R are also the same distance from zero on opposite sides, so they are also opposites.

3. Read each real-world description. Write the integer that represents the opposite of the opposite. Show your work to support your answer.

   a. A temperature rise of 20 degrees Fahrenheit

   -20 is the opposite of 20 (which is a fall in temperature).

   20 is the opposite of -20 (which is a rise in temperature).

   $-(-(20)) = 20$

   > I know that the word *rise* describes a positive integer. The opposite of a positive integer is a negative integer. The opposite of negative integer is a positive integer.

   b. A loss of 15 pounds

   15 is the opposite of -15 (which is a gain of pounds).

   -15 is the opposite of 15 (which is a loss of pounds).

   $-(-(-15)) = -15$

   > I know that the word *loss* describes a negative integer. The opposite of a negative integer is a positive integer. The opposite of a positive integer is a negative integer.

4. Write the integer that represents the statement. Locate and label each integer on the number line below. Plot each integer with a point on the number line.

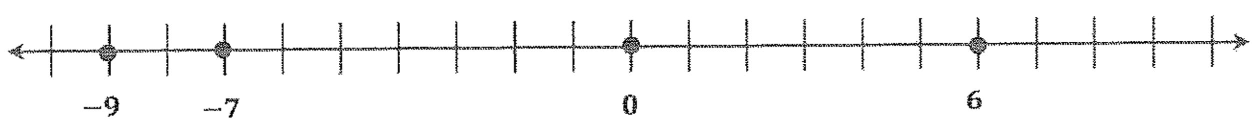
   a. The opposite of a gain of 7 **-7**

   b. The opposite of a deposit of $9 **$-9$**

   c. The opposite of the opposite of 0 **0**

   d. The opposite of the opposite of 6 **6**

   > I know that the words *gain* and *deposit* describe a positive integer.



G6-M3-Lesson 6: Rational Numbers on the Number Line

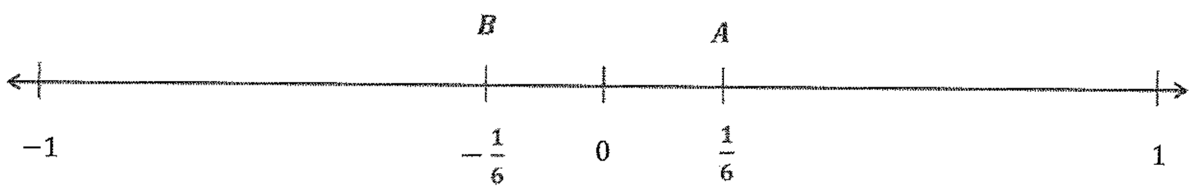
1. In the space provided, write the opposite of each number.

   a. $\frac{11}{8}$ $\qquad -\frac{11}{8}$

   b. $-\frac{7}{4}$ $\qquad \frac{7}{4}$

   c. 5.67 $\qquad -5.67$

2. Choose a non-integer between 0 and 1. Label it point A and its opposite B on the number line. Write values below the points.



I know integers are numbers that are not fractional. A non-integer is a number that can be a fraction.

   a. To draw a scale that would include both points, what could be the length of each segment?

   The length of each segment could be $\frac{1}{6}$.

   b. In words, create a real-world situation that could represent the number line diagram.

   Starting from school, the track is $\frac{1}{6}$ of a mile away. The baseball field is $\frac{1}{6}$ of a mile from the school in exactly the opposite direction.

3. Choose a value for point P that is between -8 and -9.

$-\frac{26}{3}$

I can choose any non-integer less than -8 and more than -9. This can be a fraction or a decimal.

a. What is the opposite of P?

$\frac{26}{3}$

b. Use the value from part (a), and describe its location on the number line in relation to zero.

$\frac{26}{3}$ is the same distance as $-\frac{26}{3}$ from zero but to the right. $\frac{26}{3}$ is $8\frac{2}{3}$ units to the right of (or above) zero.

c. Find the opposite of the opposite of point P. Show your work and explain your reasoning.

The opposite of the opposite of the number is the original number. If P is located at $-\frac{26}{3}$, then the opposite of the opposite of P is located at $-\frac{26}{3}$. The opposite of $-\frac{26}{3}$ is $\frac{26}{3}$. The opposite of $\frac{26}{3}$ is $-\frac{26}{3}$. $-\left(-\left(-\frac{26}{3}\right)\right) = -\frac{26}{3}$

4. Locate and label each point on the number line. Use the diagram to answer the questions.

Ami lives one block north of the hair salon.

Trisha's house is $\frac{1}{4}$ of a block past Ami's house.

Isa and Shane are at the soccer field $\frac{6}{4}$ blocks south of the hair salon.

The grocery store is located halfway between the hair salon and the soccer field.

I know that each of the values in the problem has a denominator of 4, so I separated my number line into equal units of $\frac{1}{4}$. From there, I know one whole is $\frac{4}{4}$ to locate Ami's house.

a. Describe an appropriate scale to show all the points in the situation.

An appropriate scale would be $\frac{1}{4}$ because the numbers given in the example all have denominators of 4. I would divide the number line by equal segments of $\frac{1}{4}$.

b. What number represents the location of the grocery store? Explain your reasoning.

The number is $-\frac{3}{4}$. I found the location of the soccer field, which is 6 units below zero. Half of 6 is 3, so I moved down 3 units from zero.

G6-M3-Lesson 7: Ordering Integers and Other Rational Numbers

1. In the table below, list each set of rational numbers in order from least to greatest. Then, list their opposites. Finally, list the opposites in order from least to greatest.

Rational Numbers	Ordered from Least to Greatest	Opposites	Opposites Ordered from Least to Greatest
$-6.1, -6.35$	$-6.35, -6.1$	$6.35, 6.1$	$6.1, 6.35$
$\frac{1}{3}, -\frac{1}{4}$	$-\frac{1}{4}, \frac{1}{3}$	$\frac{1}{4}, -\frac{1}{3}$	$-\frac{1}{3}, \frac{1}{4}$
$-49.9, -50$	$-50, -49.9$	$50, 49.9$	$49.9, 50$
$32\frac{1}{3}, 32$	$32, 32\frac{1}{3}$	$-32, -32\frac{1}{3}$	$-32\frac{1}{3}, -32$
$65.03, 65.05$	$65.03, 65.05$	$-65.03, -65.05$	$-65.05, -65.03$

I can visualize a number line to order the rational numbers from least to greatest. The number farthest to the left on the number line is the least, and the number to the right is the greatest.

2. For each row, what pattern do you notice between the numbers in the second and fourth columns? Why is this so?

For each row, the numbers in the second and fourth columns are opposites, and their order is opposite. This is because on the number line, as you move to the right, numbers increase. But as you move to the left, numbers decrease.

G6-M3-Lesson 8: Ordering Integers and Other Rational Numbers

1. In the table below, list each set of rational numbers in order from greatest to least. Then, in the appropriate column, state which number was farthest right and which number was farthest left on the number line.

Column 1	Column 2	Column 3	Column 4
Rational Numbers	**Ordered from Greatest to Least**	**Farthest Right on the Number Line**	**Farthest Left on the Number Line**
$-2.85, -4.15$	$-2.85, -4.15$	-2.85	-4.15
$\frac{1}{3}, -3$	$\frac{1}{3}, -3$	$\frac{1}{3}$	-3
$0.04, 0.4$	$0.4, 0.04$	0.4	0.04
$0, -\frac{1}{3}, -\frac{2}{3}$	$0, -\frac{1}{3}, -\frac{2}{3}$	0	$-\frac{2}{3}$

I can visualize a number line to order the rational numbers from greatest to least. The number farthest to the right on the number line is the greatest. The number farthest to the left is the least number.

a. For each row, describe the relationship between the number in Column 3 and its order in Column 2. Why is this?

The number in Column 3 is the first number listed in Column 2. Since it is the farthest right number on the number line, it will be the greatest; therefore, it comes first when ordering the numbers from greatest to least.

b. For each row, describe the relationship between the number in Column 4 and its order in Column 2. Why is this?

The number in Column 4 is the last number listed in Column 2. Since it is farthest left on the number line, it will be the least; therefore, it comes last when ordering from greatest to least.

2. If two rational numbers, a and b, are ordered such that a is less than b, then what must be true about the order of their opposites: $-a$ and $-b$?

   The order will be reversed for the opposites, which means $-a$ is greater than $-b$.

3. Read each statement, and then write a statement relating the *opposites* of each of the given numbers.

   a. 8 is greater than 7.

   -8 is less than -7.

   b. 48.1 is greater than 40.

   -48.1 is less than -40.

   I notice that the order is reversed for the opposites.

   c. $-\frac{1}{2}$ is less than $-\frac{1}{6}$.

   $\frac{1}{2}$ is greater than $\frac{1}{6}$.

4. Order the following from least to greatest: $-8, -17, 0, \frac{1}{2}, \frac{1}{4}$.

   $\mathbf{-17, -8, 0, \frac{1}{4}, \frac{1}{2}}$

When I order from least to greatest, I think about the number that is farthest left on the number line. When I order from greatest to least, I start with the number farthest to the right on the number line.

5. Order the following from greatest to least: $-14, 14, -20, 2\frac{1}{2}, 7$.

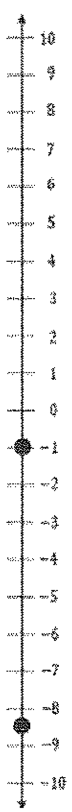
   $\mathbf{14, 7, 2\frac{1}{2}, -14, -20}$

G6-M3-Lesson 9: Comparing Integers and Other Rational Numbers

Write a story related to the points shown in each group. Be sure to include a statement relating the numbers graphed on the number line to their order.

1. 

Julia did not improve on her Sprint yesterday. Today, she improved her score by three points. Zero represents earning no improvement points yesterday, and 3 represents earning 3 improvement points. Zero is graphed to the left of 3 on the number line. Zero is less than 3.

2. 

A turtle is swimming one foot below the surface of the water. An eel is swimming $8\frac{1}{2}$ feet below the water's surface. $-8\frac{1}{2}$ is farther below zero than -1, so the eel is swimming deeper than the turtle.

I know that as numbers are farther down a vertical number line, the values of the numbers decrease. The greater of two numbers is the number that is farthest up.

G6-M3-Lesson 10: Writing and Interpreting Inequality Statements Involving Rational Numbers

For each of the relationships described below, write an inequality that relates the rational numbers.

1. Ten feet below sea level is farther below sea level than $5\frac{1}{4}$ feet below sea level.

$$\mathbf{-10 < -5\frac{1}{4}}$$

2. Kelly's grades on her last three tests were 85, 90, and $75\frac{1}{2}$. A score of $75\frac{1}{2}$ is worse than a score of 85. A score of 85 is worse than a score of 90.

$$\mathbf{75\frac{1}{2} < 85 < 90}$$

For each of the following, use the information given by the inequality to describe the relative position of the numbers on a horizontal number line.

3. $-3.4 < 0 < 3.2$

   -3.4 is to the left of zero, and zero is to the left of 3.2; or 3.2 is to the right of zero, and zero is to the right of -3.4.

4. $-5.7 < -5\frac{1}{2} < -5$

   -5.7 is to the left of $-5\frac{1}{2}$, and $-5\frac{1}{2}$ is to the left of -5; or -5 is to the right of $-5\frac{1}{2}$, and $-5\frac{1}{2}$ is to the right of -5.7.

Fill in the blanks with numbers that correctly complete each of the statements.

5. Three integers between -5 and -1 **$-4, -3, -2$**

6. Three rational numbers between -3 and -4 **$-3.45, -3.6, -3.99$**

Any rational number between -3 and -4 is acceptable.

G6-M3-Lesson 11: Absolute Value—Magnitude and Distance

1. For the following two quantities, which has the greater magnitude? (Use absolute value to defend your answers.)

   -13.6 pounds and -13.68 pounds

   $|-13.6| = 13.6$ $|-13.68| = 13.68$

   $13.6 < 13.68$, *so* -13.68 *has the greater magnitude.*

   > I can find the absolute value of both numbers and compare. The *magnitude* of a measurement is the absolute value of its measure.

2. Find the absolute value of the numbers below.

   a. $|8| =$

   b. $|-96.2| =$

   c. $|0| =$

   > In part (a), 8 is 8 units from 0, so the absolute value of 8 is 8. -96.2 is 96.2 units from 0, so its absolute value is 96.2. The absolute value of 0 is 0 and is neither positive nor negative.

   a. $|8| = 8$

   b. $|-96.2| = 96.2$

   c. $|0| = 0$

3. Write a word problem whose solution is $|150| = 150$.

   Answers will vary. Kendra went hiking and was 150 feet above sea level.

   > If sea level is the reference point, I know a positive number (150) will represent a number above sea level, and a negative number (-80) will represent a number below sea level.

4. Write a word problem whose solution is $|-80| = 80$.

   Answers will vary. Kristen went scuba diving and was 80 feet below sea level.

G6-M3-Lesson 12: The Relationship Between Absolute Value and Order

1. Jessie and Makayla each have a set of five rational numbers. Although their sets are not the same, their sets of numbers have absolute values that are the same. Show an example of what Jessie and Makayla could have for numbers. Give the sets in order and the absolute values in order.

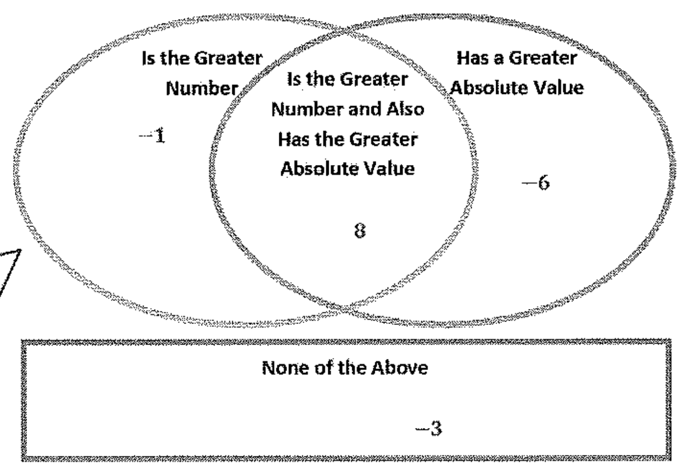
   Examples may vary. If Jessie had $2, 4, 6, 8, 10$, then her order of absolute values would be the same: $2, 4, 6, 8, 10$. If Makayla had the numbers $-10, -8, -6, -4, -2$, then her order of absolute values would also be $2, 4, 6, 8, 10$.

   Since the absolute value of a number is the distance between the number and zero on the number line, it is always a positive value. A number and its opposite have the same absolute value, so I can use any five rational numbers for Jessie's list and their opposites for Makayla's list. To put the numbers in Makayla's list in order, I remember to think of where those numbers are on the number line.

2. For each pair of rational numbers below, place each number in the Venn diagram based on how it compares to the other.

   a. $-6, -1$

   b. $8, -3$



In part (a), I know -1 is greater than -6 since it's closer to 0 on the number line. I know -6 has the greater absolute value because it has a greater distance from zero. For part (b), 8 is greater than -3 and also has the larger absolute value. I can place -3 in the *None of the Above* section since it does not fit into any of the three sections of the Venn diagram.

G6-M3-Lesson 13: Statements of Order in the Real World

1. Amy's bank account statement shows the transactions below. Write rational numbers to represent each transaction, and then order the rational numbers from greatest to least.

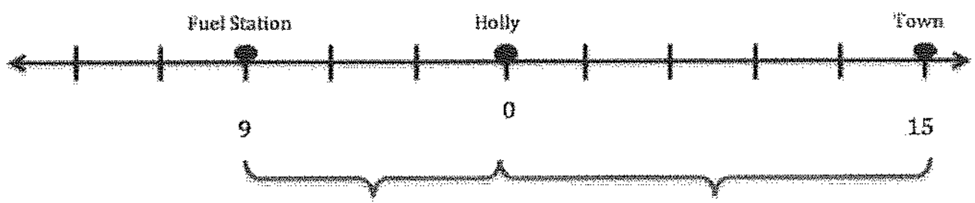
Listed Transactions	Debit $17.84	Credit $9.98	Charge $5.50	Withdrawal $35.00	Deposit $11.50	Debit $6.75	Charge $9.00
Change to Amy's Account	-17.84	9.98	-5.5	-35	11.5	-6.75	-9

$11.5 > 9.98 > -5.5 > -6.75 > -9 > -17.84 > -35$

I visualize the number line to help me determine the placement of the numbers in relation to zero.

The words "debit," "charge," and "withdrawal" all describe transactions in which money is taken out of Amy's account, decreasing its balance. I represent these transactions with negative numbers. The words "credit" and "deposit" describe transactions that will put money into Amy's account, increasing its balance, so I represent these transactions with positive numbers.

2. The fuel gauge in Holly's car says she has 29 miles to go until the tank is empty. She passed a fuel station 9 miles ago, and a sign says there is a town 15 miles ahead. If she takes a chance and drives ahead to the town and there isn't a fuel station, does she have enough fuel to go back to the fuel station? Include a diagram along a number line, and use absolute value to find your answer.



No, Holly does not have enough fuel to drive to the town and back to the gas station.

If I start at 0, where Holly is, I can think about the total number of miles from Holly to town and then how many miles it is back to the fuel station. The distance from where Holly is to town is 15 miles; then, to get to the fuel station from town, she would have to go 24 miles, which is calculated by $|15| + |-9| = 15 + 9$. The total distance is $15 + 24$, which is 39 miles. Holly would not have enough gas since she only has enough fuel for 29 miles.

She needs 15 miles worth of gas to get to town, which reduces the distance she is able to go to 14 miles ($29 - 15 = 14$). If she has to turn back and head to the fuel station, the distance is 24 miles which is calculated by $|15| + |-9| = 15 + 9$. Holly would be 10 miles short on fuel. It would be safer to go back to the fuel station without going to the town first.

G6-M3-Lesson 14: Ordered Pairs

1. Use the set of ordered pairs below to answer each question.

$$\{(6,15),(25,5),(1,2),(18,3),(2,17),(5,40),(1,7),(12,36),(0,9)\}$$

a. Write the ordered pair(s) whose first and second coordinate have a greatest common factor of 3.

$(6,15)$ *and* $(18,3)$

> I can look for ordered pairs where the first and second coordinates are multiples of 3. I can eliminate $(12,36)$ because 12 is actually the GCF of 36, not 3.

b. Write the ordered pair(s) whose first coordinate is a factor of its second coordinate.

$(1,2),(5,40),(1,7)$ *and* $(12,36)$

> I can look for ordered pairs where the first coordinate can be multiplied by a number to get the second coordinate. So I know the first coordinate in each ordered pair is a factor of its second coordinate.
>
> $1 \times 2 = 2, 5 \times 8 = 40, 1 \times 7 = 7, 12 \times 3 = 36$

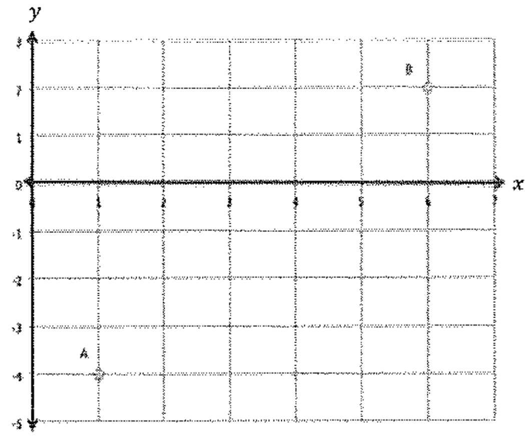
c. Write the ordered pairs(s) whose second coordinate is a prime number.

$(25,5),(1,2),(18,3),(2,17)$ *and* $(1,7)$

> I know 5, 2, 3, 17, and 7 are prime since they have exactly two factors, one and itself. In the other ordered pairs, the second coordinate is a composite number with three or more factors.

2. Write ordered pairs that represent the location of points A and B, where the first coordinate represents the horizontal direction, and the second coordinate represents the vertical direction.

   $A: (1, -4)$ $B: (6, 2)$



> I start at the origin $(0, 0)$. The first coordinate describes the location of the point using the horizontal direction, and the second coordinate describes the location of the point using the vertical direction. To get to point A, I can move 1 unit to the right and 4 units down. To get to point B, I can move 6 units to the right and 2 units up.

Extension:

3. Write ordered pairs of integers that satisfy the criteria in each part below. Remember that the origin is the point whose coordinates are $(0, 0)$. When possible, give ordered pairs such that (i) both coordinates are positive, (ii) both coordinates are negative, and (iii) the coordinates have opposite signs in either order.

   a. These points' vertical distance from the origin is twice their horizontal distance.

   Answers will vary; examples are $(1, 2)$, $(-3, 6)$, $(-2, -4)$.

> The x-coordinate (the 1st coordinate) represents the horizontal distance from the origin, and the y-coordinate (the 2nd coordinate) represents the vertical distance from the origin. Whatever distance I choose for the x-coordinate is half the distance of the y-coordinate since the vertical distance from the origin is twice the horizontal distance.

> Distance is always positive, so I know the point $(-3, 6)$ is 3 units from the origin when I count horizontally and 6 units from the origin when I count vertically. I have to remember to pay close attention to the signs and what they mean in the context of this problem.

   b. These points' horizontal distance from the origin is two units more than the vertical distance.

   Answers will vary; examples are $(7, 5)$, $(-7, 5)$, $(-7, -5)$, $(7, -5)$.

> For each ordered pair, the absolute value of the x-coordinate is 2 more than the absolute value of the y-coordinate.

   c. These points' horizontal and vertical distances from the origin are equal, but only one coordinate is positive.

   Answers will vary; examples are $(2, -2)$, $(-11, 11)$.

G6-M3-Lesson 15: Locating Ordered Pairs on the Coordinate Plane

1. Name the quadrant in which each point lies. If the point does not lie in a quadrant, specify on which axis the point lies.

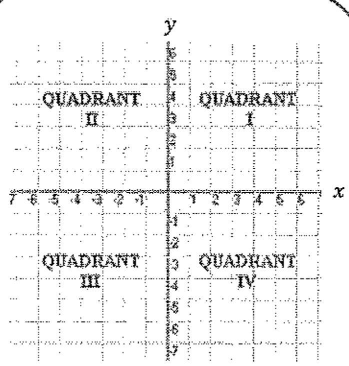
$(-1, 7.5)$
Quadrant II

$(7, -1)$
Quadrant IV

$(-6, -7)$
Quadrant III

$(2, 4)$
Quadrant I

$(0, 0)$
None; the point is not in a quadrant because it lies on the x-axis and y-axis.

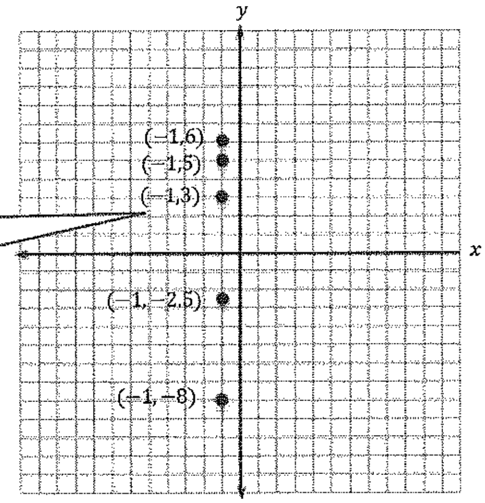


I can think about the relationship between the coordinates in each quadrant. In Quadrant I, both coordinates have positive values. In Quadrant II, the first coordinate is negative, and the second coordinate is positive. In Quadrant III, both coordinates have negative values. In Quadrant IV, the first coordinate is positive, and the second coordinate is negative.

2. Locate and label each set of points on the coordinate plane. Describe similarities of the ordered pairs in each set, and describe the points on the plane.

$\{(-1,3),(-1,5),(-1,6),(-1,-8),(-1,-2.5)\}$

The ordered pairs all have x-coordinates of -1, and the points lie along a vertical line above and below $(-1,0)$.



I notice the x-coordinates are negative and all the same, so I know all the points will fall on a vertical line to the left of $(0,0)$.

3. Locate and label at least five points on the coordinate plane that have an x-coordinate of -3.
   a. What is true of the y-coordinates below the x-axis?

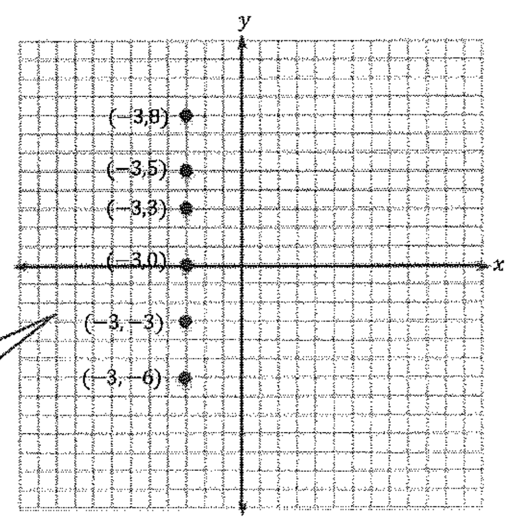
      The y-coordinates are all negative values.

   b. What is true of the y-coordinates above the x-axis?

      The y-coordinates are all positive values.

   c. What must be true of the y-coordinate on the x-axis?

      The y-coordinate on the x-axis must be 0.

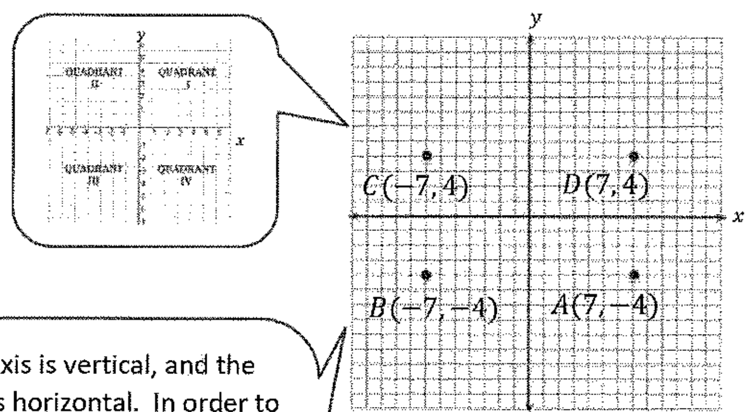


I can graph six points on the coordinate plane and look for relationships between the ordered pairs.

G6-M3-Lesson 16: Symmetry in the Coordinate Plane

1. Locate a point in Quadrant IV of the coordinate plane. Label the point A, and write its ordered pair next to it.

   Answers will vary; Quadrant IV $(7, -4)$



The y-axis is vertical, and the x-axis is horizontal. In order to reflect point A so its image is in Quadrant III, it has to be reflected over the y-axis. I notice the relationship between the signs of each coordinate.

   a. Reflect point A over an axis so that its image is in Quadrant III. Label the image B, and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points A and B?

      $B(-7, -4)$; ***reflected over the*** y***-axis***

      The ordered pairs differ only by the sign of their x***-coordinates:*** $A(7, -4)$ ***and*** $B(-7, -4)$.

b. Reflect point B over an axis so that its image is in Quadrant II. Label the image C, and write its ordered pair next to it. Which axis did you reflect over? What is the only difference in the ordered pairs of points B and C? How does the ordered pair of point C relate to the ordered pair of point A?

$C(-7, 4)$; reflected over the x-axis

The ordered pairs differ only by the signs of their y-coordinates: $B(-7, -4)$ and $C(-7, 4)$.

The ordered pair for point C differs from the ordered pair for point A by the signs of both coordinates: $A(7, -4)$ and $C(-7, 4)$.

c. Reflect point C over an axis so that its image is in Quadrant I. Label the image D, and write its ordered pair next to it. Which axis did you reflect over? How does the ordered pair for point D compare to the ordered pair for point C? How does the ordered pair for point D compare to points A and B?

$D(7, 4)$; reflected over the y-axis again

Point D differs from point C by only the sign of its x-coordinate: $D(7, 4)$ and $C(-7, 4)$.

Point D differs from point B by the signs of both coordinates: $D(7, 4)$ and $B(-7, -4)$.

Point D differs from point A by only the sign of the y-coordinate: $D(7, 4)$ and $A(7, -4)$.

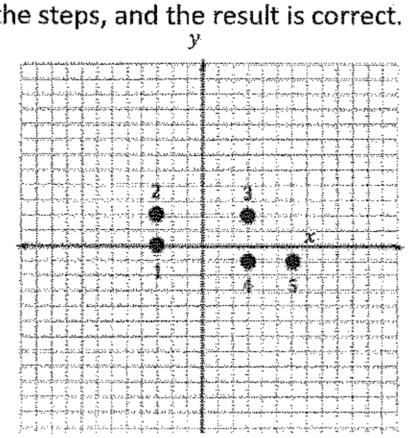
2. Trudy listened to her teacher's directions and navigated from the point $(-3, 0)$ to $(6, -1)$. She knows that she has the correct answer, but she forgot part of the teacher's directions. Her teacher's directions included the following:

"Move 2 units up, reflect about the __?__-axis, move down 3 units, and then move right 3 units."

Help Trudy determine the missing axis in the directions, and explain your answer.

The missing line is a reflection over the y-axis. The first move would move the location of the point to $(-3, 2)$ in Quadrant II. A reflection over the y-axis would move the location to $(3, 2)$ in Quadrant I. A move down 3 units and to the right 3 units would result in the end point $(6, -1)$.

I can visualize a coordinate plane or actually sketch one to help me with this problem. I graphed the first ordered pair at $(-3, 0)$ and then followed each direction. I can reflect the point located at 2 across the x-axis, but if I follow the rest of the steps, the result is not $(6, -1)$. I can reflect the point located at 2 across the y-axis, follow the steps, and the result is correct.



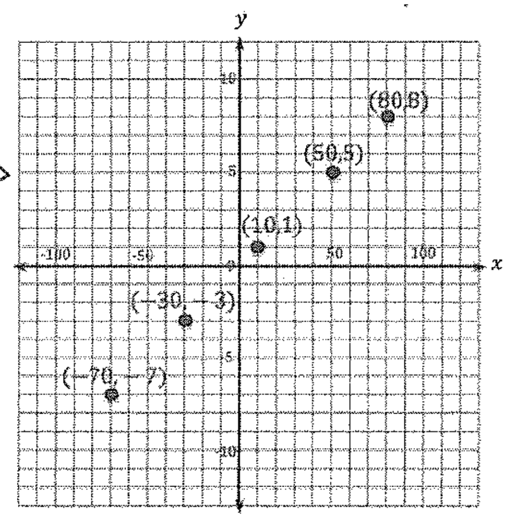
EUREKA MATH

G6-M3-Lesson 17: Drawing the Coordinate Plane and Points on the Plane

Label the coordinate plane, and then locate and label the set of points below.

$$\left\{\begin{matrix}(80,8),(50,5),(10,1),\\(-30,-3),(-70,-7)\end{matrix}\right\}$$

To label the coordinate plane, I can look at the range of numbers for the x-coordinates and y-coordinates. In the x-coordinates, the range is from -70 to 80, so I can label the x-axis from -100 to 100. For the y-coordinates, the range is from -7 to 8, so I can label the y-axis from -10 to 10. Now, I can locate and label the points listed above.



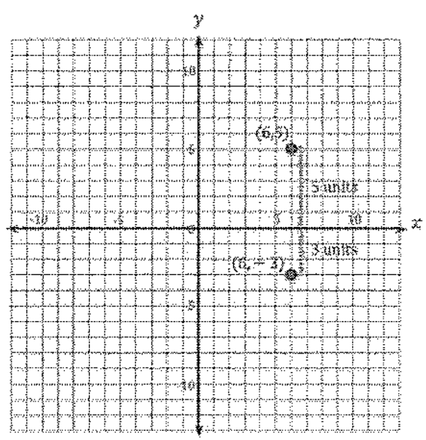
Extension:

Describe the pattern you see in the coordinates and the pattern you see in the points. Are these patterns consistent for other points, too?

The x-coordinate for each of the given points is 10 times its y-coordinate. When I graph the points, they appear to make a straight line. I check other ordered pairs with the same pattern, such as $(-10,-1)$, $(30,3)$, and even $(0,0)$, and it appears that these points are also on that line.

G6-M3-Lesson 18: Distance on the Coordinate Plane

1. Find the length of the line segment with end points $(6, 5)$ and $(6, -3)$, and explain how you arrived at your solution.



When I sketch a graph and locate the two given points, I can actually see the line segment, and it becomes easier for me to explain how to find the length.

The distance is 8 units. Both points have the same x-coordinate, so I knew they were on the same vertical line. I found the distance between the y-coordinates by counting the number of units on a vertical number line from -3 to zero and then from zero to 5, and $3 + 5 = 8$.

or

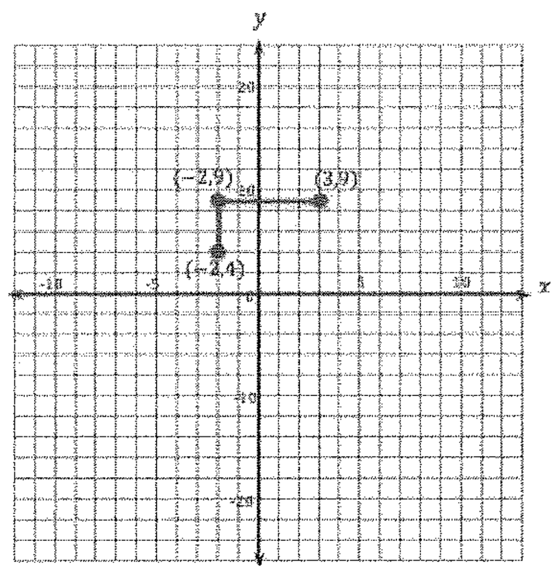
I found the distance between the y-coordinates by finding the absolute value of each coordinate. $|5| = 5$ and $|-3| = 3$. The coordinates lie on opposite sides of zero, so I found the length by adding the absolute values together. Therefore, the length of a line segment with end points $(6, 5)$ and $(6, -3)$ is 8 units.

2. Kelly and Dave were learning partners in math class and were working independently. They each started at the point $(-7, 1)$ and moved 6 units vertically in the plane. Each student arrived at a different end point. How is this possible? Explain and list the two different end points.

It is possible because Kelly could have counted up, and Dave could have counted down or vice versa. Moving 6 units in either direction vertically would generate the following possible end points: $(-7, 7)$ or $(-7, -5)$.

3. The length of a line segment is 5 units. One end point of the line segment is $(-2, 9)$. Find four points that could be the other end points of the line segment.

   $(-2, 14)$, $(-2, 4)$, $(-7, 9)$ *or* $(3, 9)$



I can sketch a graph (shown to the left) and label the given end point $(-2, 9)$. I can count 5 units to the right, and the resulting end point is $(3, 9)$. Since these two points are on a horizontal line, the y-coordinates are the same. I can count 5 units down, and the resulting end point is $(-2, 4)$. Since these two points are on a vertical line, the x-coordinates are the same. I can also count 5 units up and 5 units to the left, which are not shown, and record the resulting end points.

G6-M3-Lesson 19: Problem Solving and the Coordinate Plane

1. One end point of a line segment is $(-2, -7)$. The length of the line segment is 4 units. Find four points that could serve as the other end point of the given line segment.

   $(-2, -11)$, $(-2, -3)$, $(2, -7)$, $(-6, -7)$

   To find four points that could be other end points for the line segment, I can move vertically or horizontally from the given point, $(-2, -7)$. If I move vertically, the y-coordinate changes, but the x-coordinate stays the same. Moving 4 units up would result in an end point of $(-2, -3)$, and moving 4 units down would result in an end point of $(-2, -11)$. If I move horizontally from the given point, the x-coordinate changes, but the y-coordinate stays the same. Moving 4 units to the right would result in an endpoint of $(2, -7)$, and moving 4 units to the left would result in an end point of $(-6, -7)$.

2. Two of the vertices of a rectangle are $(3, -4)$ and $(-5, -4)$. If the rectangle has a perimeter of 24 units, what are the coordinates of its other two vertices?

   $(-5, 0)$ *and* $(3, 0)$ *or* $(-5, -8)$ *and* $(3, -8)$

   Since the two given points have the same x-coordinate, I know they are on the same horizontal line. I also know the distance from 3 to zero is 3, and the distance from zero to -5 is 5, so the total length of the line segment is 8 units. In a rectangle, there are two pairs of parallel sides, so I know the opposite side of the rectangle is also 8 units. Since the perimeter is 24 units, I can find the sum of $8 + 8$, which is 16, and subtract from 24 to determine the length of the other two sides. $24 - 16 = 8$, so the sum of the other two sides is 8. Since the remaining two sides are the same, $8 \div 2 = 4$. The length of each side is 4. From each of the given coordinates, I can count 4 units up (or down) to determine the coordinates of the other two vertices.

3. A rectangle has a perimeter of 14 units, an area of 12 square units, and sides that are either horizontal or vertical. If one vertex is the point $(-3, -2)$, and the origin is in the interior of the rectangle, find the vertex of the rectangle that is opposite $(-3, -2)$.

   $(1, 1)$

I can list the factors of 12 since the area is 12 square units. The factors are $1, 2, 3, 4, 6$, and 12. In order for the perimeter of the rectangle to be 14, the sum of each half of the rectangle must be 7. So, I can look at the factors and see that $4 + 3 = 7$. Starting at the given point, I notice that a length of 3 units will not work because the origin will not be in the interior of the rectangle, so the length of the rectangle is 4 units, and the width is 3 units. I can move 4 units to the right, which will result in a vertex $(1, -2)$. Since the length is 4 units, the width is 3 units. From the point $(1, -2)$, I can move 3 units up, and the resulting vertex, which is opposite $(-3, -2)$, is $(1, 1)$.